Analysis of a Negative Conductance Amplifier Operated with a Nonideal Circulator*

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Summary—Negative conductance amplifiers are usually operated with a circulator in order to achieve greater gain-bandwidth products and stable operation. Typical circulators differ from ideal circulators in that the forward loss between ports is not zero, and the reverse isolation between ports is not infinite. The main effects of noninfinite isolation are shown to be a modified gain-bandwidth product and a change in output admittance of the circulator output port. These effects result principally from the finite isolation between the output and amplifier ports. The main effect of incidental dissipation has previously been shown to be an increase in system noise figure.

This paper considers only the effects caused by noninfinite isolation. A model of a lossless three-port circulator with noninfinite isolation is set up, and a negative conductance amplifier is considered to be connected to one port of this circulator. The magnitude of negative conductance is assumed to be limited to ensure a positive output conductance at the output port of the circulator (that is, the combination of negative conductance amplifier and nonideal circulator is assumed to be open-circuit stable). Subject to this assumption, the combination of negative conductance amplifier and nonideal circulator is then analyzed for its output admittance, available power gain, and effective input noise temperature.

Introduction

The gain-bandwidth product of a one-port negative conductance amplifier (such as a maser or reactance amplifier) is greatly increased by operation with a nonreciprocal device, in particular an ideal circulator. In addition, constancy of output admittance is also greatly increased. The question arises, what is the effect on operation if the circulator is not ideal, that is, the forward loss between ports is not zero and the reverse isolation between ports is not infinite? The main effects of noninfinite isolation will be shown to be a modified gain-bandwidth product and a change in output admittance of the circulator output port. These effects result principally from the finite isolation between the output and amplifier ports. The main effect of incidental dissipation has previously been shown to be an increase in system noise figure.[1]

This paper considers only the effects caused by noninfinite isolation. More complete models of a nonideal circulator can be formulated, if desired, by using an analysis similar to that presented here. It is believed, however, that the inclusion of small forward loss will have a direct effect only on the system noise figure while not appreciably affecting the output admittance or available power gain.

A three-port circulator is considered here, with the direction of power flow from port 1 (input) to port 2 (connected to the negative conductance amplifier) to port 3 (output). Specifically, a noninfinite isolation is assumed to exist between ports 3 and 2. No other departures from ideal characteristics are assumed except those resulting from the assumption of the circulator being lossless. Some workers[1,2] analyze the effect of this noninfinite isolation by a combination of Friis's classical treatment of the over-all noise figure of cascaded networks[3] and a consideration that noise power radiates back from the input terminals of the second stage to be amplified in the first stage. This is really not a rigorous application of Friis's work. Consider the following two possible cases.

The first possibility is that the first stage consisting of the negative conductance amplifier and the nonideal circulator has a positive output conductance (that is, the first stage is open-circuit stable). Then, as discussed by Friis, both the available power gain and the effective input noise temperature[4] of the first stage can be calculated for a particular generator admittance without specifying any characteristics of the second stage. The noise emanating from the input terminals of the second stage, if any, will affect only the effective input noise temperature of the second stage, which must be measured using a source admittance equal to the output admittance of the first stage. For this case, the over-all effective input noise temperature T can be expressed in terms of the first stage effective input noise temperature T_1, the first stage available power gain K_1, and the second stage effective input noise temperature T_2 by the relation, $T=T_1+T_2/K_1$. This assumes that subsequent stages do not contribute to the value of T. If they do, and if each subsequent stage has a positive output conductance, terms of the form $T_n/K_1K_2 \cdots K_{n-1}$, where n denotes the stage number, can be added to the relation for T.

The second possibility is that the first stage, consisting of the negative conductance amplifier and the nonideal circulator, has a negative output conductance. Then, to avoid oscillations, the input conductance of the second stage must be sufficiently positive to make the net conductance positive at the junction of the first and second stages. This possibility is actually more likely to

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[1] F. R. Arams and G. Krayer, "Design considerations for circulator maser systems," Proc. IRE, vol. 46, pp. 912–913; May, 1958.

[2] A. E. Siegman, "Gain bandwidth and noise in maser amplifiers," Proc. IRE, vol. 45, pp. 1737–1738; December, 1957.

[3] H. T. Friis, "Noise figures of radio receivers," Proc. IRE, vol. 32, pp. 419–422; July, 1944.

[4] Friis uses noise figure rather than effective input noise temperature, but this does not affect the argument, since the two quantities are directly related.

occur when operating a negative conductance amplifier without a nonreciprocal device. Here, very definitely, Friis's definitions do not directly apply since his concepts of available signal and noise power outputs are meaningless. In this case, it does seem reasonable to think of the noise from the input terminals of the second stage radiating back to the first stage to be amplified. Now, however, the characteristics of the second stage must be specified to describe the performance of the first stage. Furthermore, how does one calculate simply the contribution of the second stage to the over-all effective input noise temperature? The usual expedient is to assume that the first stage gain (not available gain) is so large that the second stage contribution is negligible. This may not be a good assumption in practice, however, when dealing with an extremely low noise first stage or with a second stage whose noisiness is critically dependent on its source admittance. In any case, from a theoretical standpoint it is desirable to have a rigorous method of calculating the contributions of the second stage and subsequent stages to the over-all effective input noise temperature. Only in this way can the importance of these contributions be quantitatively assessed.

Returning now to the approach taken in this paper, it seemed desirable to apply the classical treatment of Friis to the new situation of a negative conductance amplifier. This, clearly, can be done for the combination of the negative conductance amplifier and the nonideal circulator if the assumption is made that the output conductance is positive. This is a reasonable assumption if the circulator is well made and the gain is not too high. The inequality following (17) gives the quantitative condition required to satisfy this assumption.

It is then shown, using standard network analysis, that the finite isolation of the circulator principally affects the output admittance and available power gain of the combination of negative conductance amplifier and circulator. The effective input noise temperature, however, is but slightly affected.

Admittance Matrix of Nonideal Three-port Circulator

The first step in the analysis is to determine the admittance matrix of the assumed model of a nonideal three-port circulator. A convenient way to do this is to make use of the scattering matrix concept.[5] Thus, a scattering matrix of a lossless three-port circulator with insertion power gain ϵ^2 between ports 3 and 2 is

$$S = \begin{bmatrix} j\epsilon & 0 & \sqrt{1-\epsilon^2} \\ \sqrt{1-\epsilon^2} & 0 & j\epsilon \\ 0 & 1 & 0 \end{bmatrix}. \tag{1}$$

[5] H. J. Carlin, "The scattering matrix in network theory," IRE Transactions on Circuit Theory, vol. CT-3, pp. 88–97; June, 1956.

Some discussion of (1) is in order. In general, the insertion power gain between ports m and n of a network is $|s_{n,m}|^2$ (note the interchange in order of subscripts). Thus, having $s_{23}=j\epsilon$ in (1) provides an insertion power gain ϵ^2 between ports 3 and 2, as desired. The assumed phase of s_{23}, as well as the particular nonideal values of s_{11}, s_{13}, and s_{21}, seem to be the simplest way of maintaining the assumed lossless character of the circulator, the condition for which is $[S^{*T}][S]=E$, where $[S^{*T}]$ is the matrix transpose of the complex conjugate of S, and E is the identity matrix.

If the currents and voltages at all three ports are normalized to the same conductance G_0, the admittance matrix is then

$$Y = G_0[E+S]^{-1}[E-S]. \tag{2}$$

Finally, substitution of (1) into (2) gives, for the admittance matrix of the nonideal three-port circulator,

$$\begin{aligned} Y &= G_0\begin{bmatrix} -j\epsilon & \sqrt{1-\epsilon^2} & -\sqrt{1-\epsilon^2} \\ -\sqrt{1-\epsilon^2} & j\epsilon & 1-j\epsilon \\ \sqrt{1-\epsilon^2} & -(1+j\epsilon) & j\epsilon \end{bmatrix} \\ &= \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}. \end{aligned} \tag{3}$$

As a sidelight, substitution of $\epsilon=0$ in (3) gives the admittance matrix of an ideal three-port circulator;

$$[Y]_{\text{ideal}} = G_0\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}. \tag{4}$$

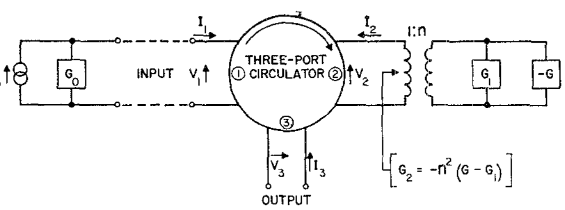

Fig. 1—Small-signal circuit of negative conductance amplifier-circulator combination.

Output Admittance of Negative Conductance Amplifier with Nonideal Circulator

Fig. 1 shows the circuit of the negative conductance amplifier-circulator combination. The negative conductance amplifier is represented by a negative conductance $-G$, a circuit loss G_1, and an ideal transformer of turns ratio $1:n$. Also included are a current generator I_G and its associated conductance G_0. It is seen that this is a midband representation of the negative conductance amplifier, since no susceptances are shown.

The terminal currents and voltages at the three ports of the circulator are related by

$$
\begin{aligned}
I_1 &= Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 \\
I_2 &= Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 \\
I_3 &= Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3
\end{aligned}
\tag{5}
$$

where the nine Y-values are given by (3). Also, the current and voltage at port 2 are related by

$$
I_2 = -G_2V_2 \tag{6}
$$

where

$$
G_2 = -n^2(G - G_1).
$$

Substitution of (6) in (5) gives the equations of the two-port combination of negative conductance amplifier and nonideal circulator,

$$
\begin{aligned}
I_1 &= Y_{11}'V_1 + Y_{13}'V_3 \\
I_3 &= Y_{31}'V_1 + Y_{33}'V_3
\end{aligned}
\tag{7}
$$

where

$$
\begin{aligned}
Y_{11} &= Y_{11} - \frac{Y_{12}Y_{21}}{Y_{22} + G_2} \\
Y_{13}' &= Y_{13} - \frac{Y_{12}Y_{23}}{Y_{22} + G_2} \\
Y_{31}' &= Y_{31} - \frac{Y_{21}Y_{32}}{Y_{22} + G_2} \\
Y_{33}' &= Y_{33} - \frac{Y_{23}Y_{32}}{Y_{22} + G_2}.
\end{aligned}
$$

Substitution of (3) into (7) gives the normalized admittances,

$$
\begin{aligned}
y_{11}' &= \frac{Y_{11}'}{G_0} = \frac{1 - j\epsilon g_2}{g_2 + j\epsilon} \\
y_{13}' &= \frac{Y_{13}'}{G_0} = \frac{-(1 + g_2)\sqrt{1 - \epsilon^2}}{g_2 + j\epsilon} \\
y_{31}' &= \frac{Y_{31}'}{G_0} = \frac{-(1 - g_2)\sqrt{1 - \epsilon^2}}{g_2 + j\epsilon} \\
y_{33}' &= \frac{Y_{33}'}{G_0} = \frac{1 + j\epsilon g_2}{g_2 + j\epsilon}
\end{aligned}
\tag{8}
$$

where $g_2 = G_2/G_0$. The normalized output admittance is then

$$
\begin{aligned}
y_{out} &= \frac{Y_{out}}{G_0} = y_{33}' - \frac{y_{13}'y_{31}'}{y_{11}' + 1} \\
&= \frac{1}{G_0}[G_{out} + jB_{out}].
\end{aligned}
\tag{9}
$$

Finally, substitution of (8) into (9) gives the normalized output conductance and susceptance

$$
g_{out} = \frac{G_{out}}{G_0} = \frac{1 - \epsilon^2\left(\frac{1 - g_2}{1 + g_2}\right)^2}{1 + \epsilon^2\left(\frac{1 - g_2}{1 + g_2}\right)^2} \tag{10a}
$$

$$
b_{out} = \frac{B_{out}}{G_0} = \frac{-2\epsilon\left(\frac{1 - g_2}{1 + g_2}\right)}{1 + \epsilon^2\left(\frac{1 - g_2}{1 + g_2}\right)^2}. \tag{10b}
$$

Note that, since a positive value of G_{out} is assumed, $|g_2| < (1-\epsilon)/(1+\epsilon)$. Furthermore, the magnitude of output admittance is constant and equal to G_0. In terms of the normalized conductance and susceptance, $g^2_{out} + b^2_{out} = 1$. Further discussion of the variations of g_{out} and b_{out} with the value of ϵ will be postponed until the available power gain is calculated.

Available Power Gain of Negative Conductance Amplifier with Nonideal Circulator

The available power output (that is, the power that would be delivered to a conjugate load, $G_{out} - jB_{out}$) is

$$
[P_3]_{av} = \frac{|I_3|^2_{V_3=0}}{4G_{out}} \tag{11a}
$$

where $[I_3]_{V_3=0}$ is the current that would flow from the output port if it were short-circuited; and similarly, the available power input is

$$
[P_G]_{av} = \frac{|I_G|^2}{4G_0}. \tag{11b}
$$

The quotient of (11a) and (11b) gives the available power gain

$$
K_{av} = \frac{[P_3]_{av}}{[P_G]_{av}} = \frac{1}{g_{out}}\left|\frac{I_3}{I_G}\right|^2_{V_3=0}. \tag{12}
$$

Substitution of the terminal conditions, $I_1 = I_G - G_0V_1$ and $V_3 = 0$, into (7) gives the ratio of short-circuit output current to generator current

$$
\left[\frac{I_3}{I_G}\right]_{V_3=0} = \frac{Y_{31}'}{Y_{11}' + G_0} = \frac{y_{31}'}{y_{11}' + 1}. \tag{13}
$$

Finally, substitution of (13), (8), and (10a) into (12) gives, for the available power gain,

$$
K_{av} = \frac{(1 - \epsilon^2)\left(\frac{1 - g_2}{1 + g_2}\right)^2}{1 - \epsilon^2\left(\frac{1 - g_2}{1 + g_2}\right)^2}. \tag{14}
$$

From (14), if an ideal circulator were used ($\epsilon = 0$), the available power gain would be

$$
K_{av}' = \left[\frac{1 - g_2}{1 + g_2}\right]^2. \tag{15}
$$

To see the effect of ϵ on the available power gain, substitution of (15) into (14) gives the ratio of available power gains with nonideal and ideal circulators:

$$\frac{K_{av}}{K_{av}'} = \frac{1-\epsilon^2}{1-\epsilon^2 K_{av}'} \approx \frac{1}{1-\epsilon^2 K_{av}'}, \qquad \epsilon^2 \ll 1. \tag{16}$$

This is an interesting result; the available power gain is increased by noninfinite isolation between ports 3 and 2 of the circulator. Note that, neglecting any bandwidth restriction at port 3, the bandwidths are the same for these two gains, since G_2, the net negative conductance connected to port 2, is held fixed for the comparison of gains. Eq. (16) is plotted in Fig. 2, with parameter $\epsilon^2 K_{av}'$ as the abscissa.

With the help of the above expressions involving available power gains, (10) for the normalized output conductance and susceptance can be interpreted more readily. Substitution of (15) into (10) gives, for the magnitudes of the normalized output conductance and susceptance,

$$g_{out} = \frac{1-\epsilon^2 K_{av}'}{1+\epsilon^2 K_{av}'} \tag{17a}$$

$$|b_{out}| = \frac{2\sqrt{\epsilon^2 K_{av}'}}{1+\epsilon^2 K_{av}'}. \tag{17b}$$

[The negative sign of b_{out} in (10b) has no particular significance, arising as it does from the assumed positive sign of ϵ in (1).] It is seen that the output conductance decreases, and the output susceptance increases, as ϵ increases. Furthermore, the assumption of $G_{out}>0$ requires $\epsilon^2 K_{av}'<1$. Eqs. (17) are also plotted in Fig. 2.

It may be more convenient to have the normalized output conductance and susceptance related directly to the available power gain, rather than to have these three quantities each related to the available power gain if the circulator were ideal. Substitution of K_{av}' from (16) into (17) gives, for the magnitudes of the normalized output conductance and susceptance, in terms of the available power gain,

$$g_{out} = \frac{1-\epsilon^2}{1-\epsilon^2+2\epsilon^2 K_{av}} \approx \frac{1}{1+2\epsilon^2 K_{av}}, \qquad \epsilon^2 \ll 1 \tag{18a}$$

$$|b_{out}| = \frac{2\sqrt{\epsilon^2 K_{av}(1-\epsilon^2+\epsilon^2 K_{av})}}{1-\epsilon^2+2\epsilon^2 K_{av}} \approx \frac{2\sqrt{\epsilon^2 K_{av}(1+\epsilon^2 K_{av})}}{1+2\epsilon^2 K_{av}}, \qquad \epsilon^2 \ll 1. \tag{18b}$$

Eqs. (18) are plotted in Fig. 3, with parameter $\epsilon^2 K_{av}$ as the abscissa.

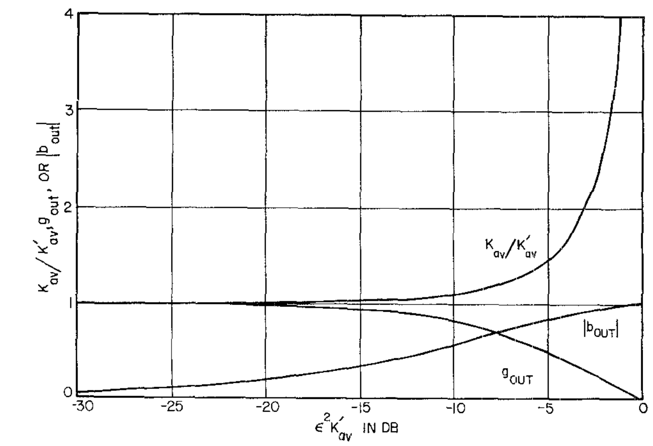

Fig. 2—Normalized gain and output admittance of negative conductance amplifier with nonideal circulator vs parameter $\epsilon^2\kappa'_{av}$.

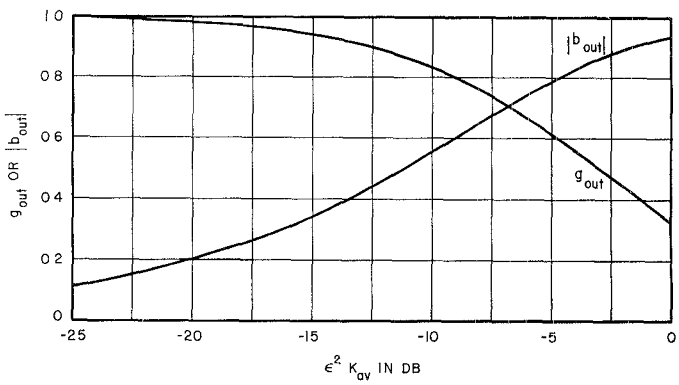

Fig. 3—Normalized output admittance of negative conductance amplifier with nonideal circulator vs parameter $\epsilon^2\kappa_{av}$.

An alternative way of describing the output admittance is in terms of the reflection coefficient and standing-wave ratio, using G_0 as the reference conductance. Thus, from (10) and (17) the output reflection coefficient is

$$\Gamma = \frac{1-y_{out}}{1+y_{out}} = -j\sqrt{\epsilon^2 K_{av}'}, \tag{19a}$$

and the output standing-wave ratio is

$$\text{SWR} = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+\sqrt{\epsilon^2 K_{av}'}}{1-\sqrt{\epsilon^2 K_{av}'}}. \tag{19b}$$

Eqs. (19) can also be expressed in terms of the actual available power gain (K_{av}) by substitution of K_{av}' from (16);

$$\Gamma = -j\sqrt{\frac{\epsilon^2 K_{av}}{1-\epsilon^2+\epsilon^2 K_{av}}} \approx -j\sqrt{\frac{\epsilon^2 K_{av}}{1+\epsilon^2 K_{av}}}, \qquad \epsilon^2 \ll 1, \tag{20a}$$

$$\text{SWR} = \frac{[\sqrt{1-\epsilon^2+\epsilon^2 K_{av}}+\sqrt{\epsilon^2 K_{av}}]^2}{1-\epsilon^2} \approx [\sqrt{1+\epsilon^2 K_{av}}+\sqrt{\epsilon^2 K_{av}}]^2, \quad \epsilon \ll 1. \tag{20b}$$

To summarize, when the product of the reverse insertion power gain from port 3 to port 2 of the actual circulator (ϵ^2) and the available power gain if the circulator were ideal (K_{av}') is not very small, compared to unity, both the available power gain and the output admittance of the negative conductance amplifier-circulator combination are greatly affected. That this product should affect operation when it is not very small compared to unity is not really surprising. The situation resembles an amplifier with positive feedback whose $\mu\beta$ product is not very small compared to unity.

Effective Input Noise Temperature of Negative Conductance Amplifier with Nonideal Circulator

For calculating the effective input noise temperature, the only source of noise considered in the negative conductance amplifier is that due to loss conductance G_1 (see Fig. 1). (There is no source of noise in the circulator since it is assumed lossless.) The value of the associated mean square noise current generator, as transformed through the ideal transformer to port 2 of the circulator, is

$$\overline{i_1^2} = 4n^2 G_1 k T_1 B \tag{21}$$

where k = Boltzmann's constant, T_1 = absolute temperature of G_1, and B = spot bandwidth for which calculation is made. Consideration of this one noise source will enable a determination to be made of the effect, if any, of the nonideal circulator.

It can be shown that the effective input noise temperature is, in general, given by

$$T_e = (F - 1)T_0 = \frac{[N_0 - K_{av}kT_0B]}{K_{av}kB} \tag{22}$$

where F = noise figure, T_0 = reference temperature of generator = 290°K, and N_0 = available noise output power. The available noise output power exclusive of the generator contribution is

$$[N_0 - K_{av}kT_0B] = \frac{[\overline{i_{out}^2}]_{V_3=0}}{4G_{out}} \tag{23}$$

where $[\overline{i_{out}^2}]_{V_3=0}$ is the short-circuit mean square noise current at the output port due to loss conductance G_1. The value of $[\overline{i_{out}^2}]_{V_3=0}$ is

$$[\overline{i_{out}^2}]_{V_3=0} = \overline{i_1^2}\left|\frac{I_3}{I_{G2}}\right|^2_{V_3=0} \tag{24}$$

where $[I_3/I_{G2}]_{V_3=0}$ is the ratio of short-circuit current at the output port (port 3 of the circulator) to a current generator I_{G2} connected to port 2 of the circulator. Substitution of (23), (24), and (21) into (22) gives, for the effective input noise temperature,

$$T_e = \left[\frac{1}{g_{out}}\right]\left[\frac{1}{K_{av}}\right]\left|\frac{I_3}{I_{G2}}\right|^2_{V_3=0}\left[\frac{n^2G_1}{G_0}\right]T_1. \tag{25}$$

The ratio $[I_3/I_{G2}]_{V_3=0}$ can be evaluated by substituting the terminal conditions $I_1 = -G_0V_1$, $I_2 = I_{G2} - G_2V_2$, and $V_3 = 0$ into (5). Then,

$$\left[\frac{I_3}{I_{G2}}\right]_{V_3=0} = \frac{Y_{32}(Y_{11} + G_0) - Y_{12}Y_{31}}{(Y_{11} + G_0)(Y_{22} + G_2) - Y_{12}Y_{21}}. \tag{26}$$

After substitution of the Y-values from (3), (26) reduces to

$$\left[\frac{I_3}{I_{G2}}\right]_{V_3=0} = -\left[\frac{2}{1 + g_2}\right]\left[\frac{1}{1 + j\epsilon\left(\frac{1 - g_2}{1 + g_2}\right)}\right]. \tag{27}$$

Finally, substitution of (10a), (14), and (27) into (25) gives

$$T_e = \left[\frac{1}{1 - \epsilon^2}\right]\left[\frac{2}{1 - g_2}\right]^2\left[\frac{n^2G_1}{G_0}\right]T_1. \tag{28}$$

It is seen that the last two factors of (28) represent the effective input noise temperature of the negative conductance amplifier alone. Thus the first two factors, since they are always greater than unity, represent a degradation factor due to use of the circulator. Unlike the available power gain and output admittance, however, the effective input noise temperature is approximately the same with ideal or nonideal circulators, if $\epsilon^2 \ll 1$.

To further interpret (28), substitution of g_2 from (15) gives for the degradation factor,

$$\left[\frac{1}{1 - \epsilon^2}\right]\left[\frac{2}{1 - g_2}\right]^2 = \left[\frac{1}{1 - \epsilon^2}\right]\left[1 + \frac{1}{\sqrt{K_{av}'}}\right]^2. \tag{29a}$$

Alternatively, substitution of K_{av}' from (16) into (29a) gives

$$\left[\frac{1}{1 - \epsilon^2}\right]\left[\frac{2}{1 - g_2}\right]^2 = \left[\frac{1}{1 - \epsilon^2}\right]\left[1 + \sqrt{\epsilon^2 + \frac{1 - \epsilon^2}{K_{av}}}\right]^2. \tag{29b}$$

Eq. (29b) shows that, for the case of high available power gain and low reverse insertion gain, the degradation factor is but slightly greater than unity.

Performance of Over-all Amplifier

It has been shown that noninfinite isolation of the circulator affects the characteristics of the combination of a negative conductance amplifier and a circulator. The output admittance (Y_{out}) and the available power gain (K_{av}) can be greatly affected, whereas the effective input noise temperature (T_e) is usually but slightly affected. The above amplifier, being a low-noise amplifier, would naturally be used as the first stage of an amplifying system. What, then, will be the effect of the above changes in characteristics of such a first stage on the effective input noise temperature of the over-all amplifier? In the introduction it was pointed out that the over-all effective input noise temperature could be expressed in terms of the characteristics of the first and second stages by the relation $T = T_1 + T_2/K_1$, where T_1 and K_1 are identified as T_e and K_{av} of the main text. The apparent effect of the nonideal circulator is then to increase K_1 and probably also to increase T_2, while leaving T_1 essentially unchanged. The increase in T_2 arises if it is assumed, as is usual, that the second stage has been designed for optimum noise performance with a G_0-generator. Without specifying how T_2 changes with the source admittance seen by the second stage (Y_{out} of the first stage), it is not clear how the quantity T_2/K_1, and thus T, is affected. In any case, conceptually, T_2 can be restored to optimum by inserting a lossless transformer between the first and second stages to provide the correct source admittance for the second stage. The result should be to restore the over-all effective input noise temperature to that obtainable if the circulator were ideal. In fact, the increased value of K_1 may even significantly lower the value of T_2/K_1, to give an even lower value of T.

Conclusions

An analysis of the operation of a negative conductance amplifier with a nonideal circulator, using an admittedly greatly simplified model of the circulator, has shown that the output admittance, available power gain, and effective input noise temperature are all affected by the circulator being nonideal. This effect is most pronounced on the first two characteristics, for which it is determined by the product of the reverse insertion power gain (isolation) from ports 3 to 2 of the circulator and the available power gain of the negative conductance amplifier if connected to an ideal circulator. For large products, a transformer may be required at the output port of the circulator to avoid degrading the performance of the second stage.

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